ALGEBRA

Algebraic expressions

(simplified, no =)

Order: BEDMAS

Ex) $3(x+5)+5x(x+7)$

- **Adding\Subtracting**

[We only +/- the coefficient of the like terms. NEVER change the variables & their exponents when +\-].

L we can only +/- like terms.
- same variable(s)
- same exponents for each value

L $3x, 3x^2, 5xy, 5xy^2, 3xy$

L $7x^2y, 3x^2y, -5yx^2$

- **Multiplication**

L mult. coefficients together

L variables mult.

L add exponents of same variable

Ex) $3 \cdot 5x = 15x$

$3x \cdot 5x^2$

$3x \cdot 5x \cdot x$

$15x^{1+2} = 15x^3$

- **Division**

L divide coefficients

L variables divided

L subtract the exponents of same variable

Ex) $\frac{15x}{3} = 5x$

$\frac{15x^2}{x}$

$15x^{2-1} = 15x$

$\frac{15x^3y}{5xy}$

$3x^{3-1}y^{1-1}$

$3x^2y^0 = 3x^2$

- **Distribution**

L mult. or divide all the terms in a bracket

L remember negatives CHANGE SIGN

Ex) $3(x+4)$

$3x+3\times4 = 3x+12$

$-3(x-4) = -3x^2-12x$

$-5(5x-4) = -5x+4$

$(8a^2+12ab)\div4$

$8a^2\div4+12ab\div4 = 2a^2+3ab$

$\frac{24a+18b}{3}$

$\frac{24a}{3} = \frac{18b}{3}$

$= 8a+6b$

- When you can't easily divide but have two divisions (or fractions)... COMMON DENOMINATOR.

Ex) $\frac{3(12a-4)}{5\times3} + \frac{5(4a-7)}{5\times3}$

① Common denominator

② Distribute

③ Combine like terms

$\frac{3(12a-4)+5(4a-7)}{15}$

$\frac{36a-12+20a-35}{15}$

$= \frac{56a-23}{15}$

Algebraic equasions
(solved, w\ =)

Order: SAMDEB

Ex) $3x+5x=5+3x$

• Solving steps

① Simplify both sides
② Combine like terms from both sides (move them by doing opposite operation)
③ Undo operations on the variable (SAMDEB)

Ex) $\frac{12d \cdot 3}{4 \cdot 3} + \frac{(4d+8) \cdot 2}{6 \cdot 2} = \frac{d \cdot 4}{3 \cdot 4} + \frac{(18d+1) \cdot 4}{3 \cdot 4}$

$\frac{63}{\cancel{12}} + \frac{8d+16}{\cancel{12}} = \frac{4d}{\cancel{12}} + \frac{72d+4}{\cancel{12}}$

$36d+8d+16=4d+72d+4$

$44d+16=76d+4$
$-44 \qquad -44$

$16=32d+4$
$-4 \qquad -4$

$\frac{12}{32}=\frac{32d}{32}$

$=$

$\frac{3}{8}$ or $0.375 = d$

• Steps for word problems

① Let an unknown be "x" (unknown you know least about).
② Write expressions in terms of "x" for the other unknowns.

Simplifying

③ Write an expression with your unknowns.
④ Simplify the expression

Solving

③ Write an equation with your unknowns.
④ Solve the equation (and answer the question)

Algebra vocabulary

• **variable** (x)
- Letters used to represent a value that we do not know.

• **coefficient** $(3x=3)$ $(x^3=1)$
- The number in front of the variable.

• **constant** (21)
- A number with no variable.

• **term**
- Groups of stuff.

ex) (1) (2) (3)
$3x^2+5x+8y$

• **monomial**
- An expression with one term.

• **polynomial**
- An expression with more than one term.

• **degree**
- the sum of all term's exponants

Ex: $3x^2+5x+3xy^3$ $(2+1+1+3=7)$

[A term with no variable has a degree of 0].

Proportions Review

Proportional situations can be expressed in many different ways

① Equivalent fraction

$\frac{a}{b} = \frac{c}{d}$

$a \div b = c \div d$

$a \cdot d = b \cdot c$

equivalent rates
equivalent ratios

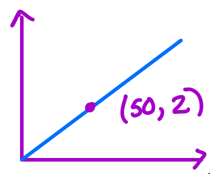
② Rule

$y = ax$

a = coefficient of proportionality

$a = \frac{y}{x}$

③ Graph
- straight line
- passes through the origin (0, 0)



to solve for a missing value in a proportional graph, you need a perfect point on the line

Ratios: $\frac{a}{b}$ or $a:b$ (a to b) or decimal $(a \div b)$

- comparison between two numbers in the <u>same type of unit</u>
- unit conversions may be necessary to get units to match

m, L or g ~~(SI units)~~

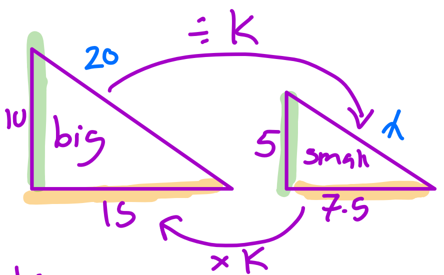
K H D M D C M ($\times 10$, $\times 10$; $\div 10$, $\div 10$)

time sec. 60 sec in min, 60 min in an hr, 3600 sec in an hr, 24 hrs in a day

Rates: $\frac{a}{b}$ or decimal $(a \div b)$

- comparison between two numbers in <u>different</u> types of units
  km/hr, \$/hr, \$/amount
- In order to solve for missing information we can find the UNIT RATE ($a \div b$ so we get num per 1 den)

Similar figures:
- corresponding angles are congruent
- corresponding sides are proportional



To solve for missing sides, we make a proportion using corresponding sides (matching sides)

$$\underset{\text{small }\Delta}{\overset{\text{big }\Delta}{}} \; \frac{10}{5} = \frac{15}{7.5} \; \underset{\leftarrow \text{small }\Delta}{\overset{\leftarrow \text{big }\Delta}{}}$$

$K = \frac{10}{5} = 2$

scale factor

* you can use the scale factor on sides or on perimeter

To solve for x

either $20 \div 2 = 10$

or $\frac{15}{7.5} = \frac{20}{x}$ $\quad x = \frac{7.5 \times 20}{1} = 10$

Solving for missing values:

Summer camps call for a 30 : 4 ratio between campers and counsellors. If the camp has hired 15 camp counsellors what is the maximum number of campers that can come?

$$\begin{array}{r} \text{campers} \rightarrow \\ \text{counsellors} \rightarrow \end{array} \frac{30}{4} = \frac{x}{15} \begin{array}{l} \leftarrow \text{new campers} \\ \leftarrow \text{counsellors} \end{array}$$

$$x = \frac{30 \times 15}{4} = 112.5$$

maximum of 112 campers (since 113 would be over ratio)

Types of Representation Review

Linear relationships always have the rule
(straight lines)

$$y = \text{pattern} \cdot x + \text{initial value}$$
$$y = ax + b$$

y: dependent variable (y depends on x)

x: independent variable

pattern: rate at which the line is going up or down $\left(\frac{\text{change in } y}{\text{change in } x}\right)$

initial value: what y is when $x=0$, where the line starts from the y-axis

Remember → proportional situations ($y=ax$) are straight lines with an initial value of 0

We can find the rule in many different ways

From a scenario:

A fishing park charges a 25\$ entrance fee as well as 2\$ per fish caught

$y = 2x + 25$

x: # of fish caught
y: total cost
(y per x)

pattern: 2 — Key words: per, every, each, /

initial value: 25 — Key words: fee, original amount, entrance cost

From a table of values:

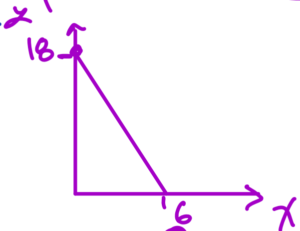
+1 +1 +1 +1

x	0	1	2	3	4
y	18	15	12	9	6

-3 -3 -3 -3

initial value
y when $x=0$

pattern $= \frac{-3}{1}$



Rule $y = -3x + 18$

or when x does NOT go up by 1

x	1	3	5	7
y	5	11	17	23

0   −1   +2   +2   +2

2 initial value   −3   +6   +6   +6

$$\text{pattern} = \frac{\text{change in } y}{\text{change in } x} = \frac{6}{2} = 3$$

initial value = go backwards in the table from y

\# in x you go back by · pattern

$1 \cdot 3$

Option 2 for initial value:

$y = 3x + b$

we can plug a point in to solve for initial value (b)

$5 = 3(1) + b$

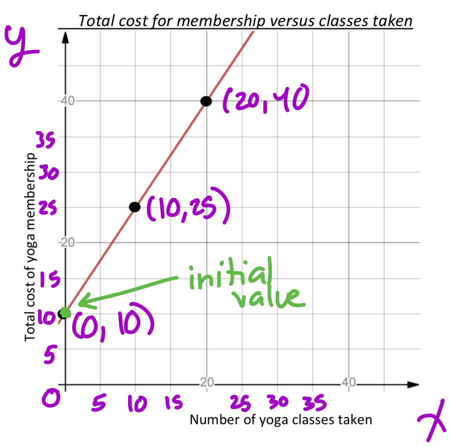
$5 = 3 + b$

$-3 \quad -3$

$2 = b$

Rule $y = 3x + 2$

From a graph



① Make a table of values with perfect points from the graph

★ pay attention to the scale of the axes

x	0	10	20
y	10	25	40

+10   +10

+15   +15

initial value is 10

$$\text{pattern} = \frac{15}{10} = 1.5 \quad \left(\frac{\text{cost}}{\text{class}} \text{ cost per class}\right)$$

Rule ⇒ $y = 1.5x + 10$

Once we have the rule we can use it to solve for missing information. ($y = 1.5x + 10$)

How much will it cost ($y=?$) for 40 yoga classes ($x=40$)?

$y = 1.5x + 10$

$y = 1.5(40) + 10$

$y = 60 + 10 = 70$

It will cost 70$

How many yoga classes ($x=?$) can you take for 56.50 $ ($y = 56.50$)

$y = 1.5x + 10$

$56.50 = 1.5x + 10$

$-10 \qquad\qquad -10$

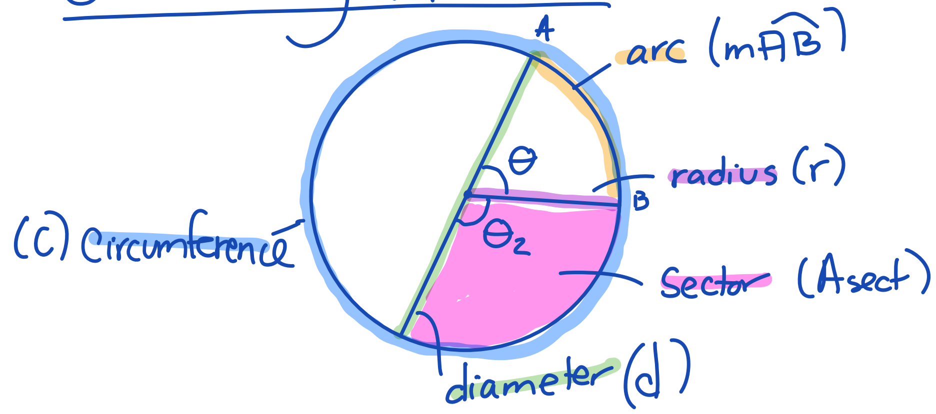
$\frac{46.50}{1.5} = \frac{1.5x}{1.5}$

$31 = x$

You can take 31 classes

Geometry Review

Circles



Formulas

$C = 2\pi r$ — If I know C, I can find d or r
or — If I know d or r I can find C
$C = \pi d$

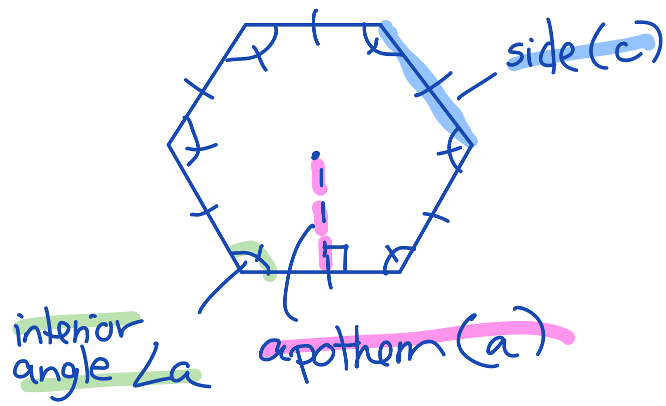
$A = \pi r^2$
— If I know r, I can find A
— If I know A, I can find r

$\frac{m\widehat{AB}}{C} = \frac{\theta}{360}$
— If I know $m\widehat{AB}$ & θ, I can find C
— If I know C & θ, I can find $m\widehat{AB}$
— If I know $m\widehat{AB}$ & C, I can find θ

$\frac{A_{sector}}{A_{circle/disc}} = \frac{\theta}{360}$
— If I know Asect & θ, I can find A
— If I know Acircle & θ, I can find Asect
— If I know Acircle & Asect, I can find θ

Regular Polygons

n # of sides = 6



Formulas

Sum of interior angles

$S = 180(n-2)$

1 interior angle

$\angle a = \dfrac{180(n-2)}{n}$

- If I know n, I can find S or ∠a
- If I know S or ∠a, I can find n

$P = n \cdot c$

- If I know n & c, I can find P
- If I know P & n, I can find c
- If I know P & c, I can find n

$A = \dfrac{nac}{2}$

or

$A = \dfrac{P \cdot a}{2}$

- If I know n, a & c, I can find A
- If I know P & a, I can find A
- If I know A, n & a, I can find c
- If I know A, n & c, I can find a
- If I know A, c & a, I can find n
- If I know A & a, I can find P

A polygon with an interior angle of 120° has a circle with an area of 254.34 cm^2 fitting perfectly inside it. What is the area of the polygon if it's side length is 12cm?

Circle

$A = 254.34 cm^2$

↳ I can find r

r = apothem

Regular polygon

$\angle a = 120°$ → I can find n

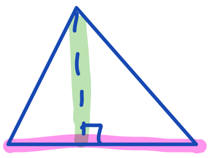
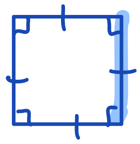
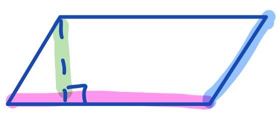
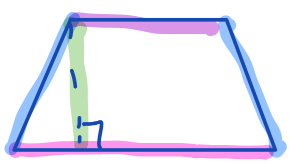
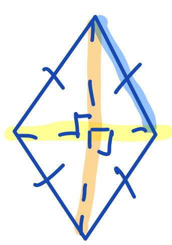
A = ?

c = 12 cm

① Find radius/apothem

$A = \pi r^2$

Extra Area Formulas

Shape		Perimeter	Area
Triangle		$P = side + side + side$	$A = \frac{b \times h}{2}$
Rectangle		$P = 2l + 2w$	$A = l \times w$
Square		$P = 4c$	$A = c^2$
Parallelogram		$P = 2b + 2side$	$A = b \times h$
Trapezoid		$P = B + b + side + side$	$A = \frac{(B+b) \times h}{2}$
Rhombus		$P = 4c$	$A = \frac{D \times d}{2}$

Percentages

$$\frac{\text{part}}{\text{total}} = \frac{\%}{100\%}$$

What is $\underbrace{15\%}_{}$ of $\underbrace{60}_{}$?

% total

$$\frac{x}{60} = \frac{15\%}{100\%} \qquad x = \frac{60 \times 15}{100} = 9$$

% total part

15% of what number is 33.75?

$$\frac{33.75}{x} = \frac{15\%}{100\%} \qquad x = \frac{33.75 \times 100}{15} = 225$$

Discount / Off / Rebate / Markdown / Less / Sale

$$\boxed{\frac{\text{final price}}{\text{original price}} = \frac{100\% - \text{discount}\%}{100\%}}$$

You paid 25$ for a shirt after a 20% discount. what was the original price of the shirt?

100-20

$$\frac{25}{\text{original}} = \frac{80}{100} \qquad x = \frac{100 \times 25}{80} = 31.25\$$$

Tax / Markup / Tip

$$\frac{\text{final price}}{\text{original price}} = \frac{100\% + \text{tax}\%}{100\%}$$

You paid $\underbrace{30\$}_{\text{final}}$ for a shirt after 15% tax. What was the original price of the shirt?

$$\frac{30}{x} = \frac{\overbrace{115}^{100+15}}{100} \qquad x = \frac{30 \times 100}{115} = 26.09\$$$

Tax & Discount

original $\xrightarrow{\text{find discount \& subtract}}$ amount with discount $\xrightarrow{\text{find tax on discounted price \& add}}$ amount with tax = final

final $\longrightarrow$ amount without tax (original without tax) $\longrightarrow$ amount without discount $\longrightarrow$ original

Dimitri pays \$27.60 for a shirt. This price includes a 20% rebate and a 15% tax. What is the regular price of the shirt?

— final price (27.60)

1) Amount paid without tax

$$\frac{\text{final}}{\text{original}} = \frac{100+\text{tax}}{100}$$

$$\frac{27.60}{x} = \frac{115}{100} \qquad x = \frac{27.60 \times 100}{115} = 24\$$$ ← price w/o tax but with the discount

2) Amount paid without discount

$$\frac{\text{final}}{\text{original}} = \frac{100-\text{discount}}{100}$$

$$\frac{24}{x} = \frac{80}{100} \qquad x = \frac{24 \times 100}{80} = 30\$$$

The regular price of the shirt is 30\$

Probability

$$\frac{\text{\# of favourable outcomes}}{\text{total \# of outcomes}}$$

Independent: One outcome does NOT depend on the previous one. They are two completely distinct experiments (coin, die) with replacement situations.

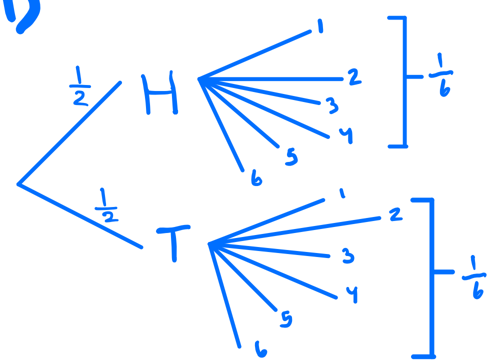
Dependent: Second outcome depends on the previous one without replacement.

When calculating probability of more than event, make a <u>tree.</u>

— — — — —

A game involves flipping a coin are once and then rolling a 6- sided die. You win the game if you flip heads and roll a number higher than 4. What is the probability of winning?

1)



$P(\text{outcome}) = P1^{st}\text{ step} \times P2^{nd}\text{ step} \ \ldots\text{etc}$

$P(H,5) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

$P(H,6) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$ ✓

$P(\text{event}) = P1^{st}\text{ outcome} \times P2^{nd}\text{ outcome} \ \ldots\text{etc}$

more than one outcome
&
more than one branch of tree